Efficient Krylov-Subspace Simulation of Autonomous RF/Microwave Circuits Driven by Digitally Modulated Carriers

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***Abstract*—The Letter presents a new approach to the simulation of self-oscillating nonlinear circuits excited by modulated RF/microwave carriers and/or dc sources. The analysis is reduced to the solution of a sequence of coupled harmonic-balance systems. A Krylov-subspace method explicitly developed for autonomous circuits and the suppression of any time-domain integration provide a dramatic speed increase with respect to conventional envelope-oriented techniques.**

***Index Terms*—Autonomous circuits and systems, envelope analysis, harmonic-balance analysis, modulators.**

I. Introduction

THE USE of hybrid time-frequency techniques is now well established for the analysis of nonlinear microwave circuits forced by digitally modulated carriers [1][2]. With these methods, baseband and microwave analysis are decoupled in the sense that the former is carried out by time-domain integration of the signal modulation laws (or complex envelopes) and the latter by performing a harmonic-balance (HB) analysis at each envelope-sampling instant. This approach thus shares some of the typical disadvantages of time-domain simulation. In addition, in the case of autonomous circuits, continuation with artificial embedding (usually implemented by introducing in the circuit a fictitious RF source or probe) [3] is normally used to solve the HB system. As a result, the analysis becomes much slower than that of a completely forced circuit with the same number of nonlinear devices, and the implementation of Krylov-subspace techniques for dealing with large circuits [4] becomes more difficult. This paper describes a Krylov-subspace algorithm, allowing large self-oscillating (autonomous) RF/microwave circuits excited by modulated carriers to be analyzed in the frequency domain by solving a sequence of modified HB systems with no time-domain integration. The phase indeterminacy of the oscillatory regime at each envelope sampling point is removed by adding an auxiliary equation enforcing the relationship between instantaneous phase and frequency deviations. In this way, most of the sophisticated computational algorithms that were previously developed for the nonautonomous case [5] can be reused for autonomous circuits, with the same efficiency in the exploitation of the available computer resources.

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II. Analysis of Autonomous Circuits Under Modulated RF Drive

The state variables (SV) of a modulated RF/microwave regime take on the form

$$\begin{aligned}\boldsymbol{x}(t) &= \sum_{\boldsymbol{k}} \boldsymbol{X}_{\boldsymbol{k}}(t)\exp(j\Omega_{\boldsymbol{k}}t)\\ &= \sum_{\boldsymbol{k}} \boldsymbol{X}_{\boldsymbol{k}}(t)\exp\left(j\sum_{h=l}^{H} k_h\omega_h t\right)\end{aligned} \tag{1}$$

where the ω_h $(1 \le h \le H)$ are RF/microwave fundamental angular frequencies (carriers), and $\boldsymbol{k}$ is an H-vector of harmonic numbers k_h. The slowly time-dependent quantities $\boldsymbol{X}_{\boldsymbol{k}}(t)$ are the complex modulation laws or time-dependent harmonics. In the forced case, the modulation laws are sampled at a number of uniformly spaced time instants $t_n (1 \le n \le N)$, called the MS instants, and the $\boldsymbol{X}_{\boldsymbol{k}}(t_n)$ are assumed as the problem unknowns. The electrical regime may be described as a sequence of RF pseudo-steady states to be determined by local HB analyses [5]. Such steady states are coupled through the envelope derivatives, appearing both in the device and in the linear subnetwork equations. In turn, the envelope derivatives may be computed by one-sided multipoint incremental rules of the form

$$\left.\frac{d\boldsymbol{X}_{\boldsymbol{k}}(t)}{dt}\right|_{t=t_n} \approx \sum_{m=0}^{M} a_m \boldsymbol{X}_{\boldsymbol{k}}(t_{n-m}) \tag{2}$$

with known coefficients a_m [6]. By requiring that the linear and nonlinear subnetwork equations be simultaneously satisfied at each MS instant, we may generate a nonlinear solving system of the form [5]

$$\boldsymbol{E}_n[\boldsymbol{S}_n; \boldsymbol{S}_{n-1}, \ldots, \boldsymbol{S}_{n-M}] = 0 \qquad (1 \le n \le N) \tag{3}$$

where $\boldsymbol{E_n}$, $\boldsymbol{S_n}$ are vectors of real and imaginary parts of the HB errors at t_n, and of the complex vector phasors $\boldsymbol{X}_{\boldsymbol{k}}(t_n)$, respectively. If we denote by P the number of positive intermodulation products $\Omega_{\boldsymbol{k}}$ of interest, (3) may be viewed as a real system of $T = n_D(2P+1)$ equations in as many unknowns (the entries of $\boldsymbol{S}_n$), with $\boldsymbol{S}_{n-1}, \ldots, \boldsymbol{S}_{n-M}$ playing the role of parameter ($n_D =$ number of device ports). Such a system is modified with respect to an ordinary HB analysis because the unknowns appear in it both in the normal way and through the expressions (2) of the envelope derivatives.

If one of the RF fundamentals, say ω_1, is autonomous, (1) cannot be directly used in the solution process because HB analysis is unable to account for the relationship between instantaneous phase and frequency deviations. Indeed, in HB terms, at each MS instant, the phase of the autonomous regime is indeterminate, and can be arbitrarily fixed. In order to circumvent this difficulty, in (1) we explicitly put into evidence the phase modulation law $\Phi_{\boldsymbol{R}}(t)$ of a reference harmonic identified by the harmonic vector $\boldsymbol{k} = \boldsymbol{R}$ with $R_1 = 1$. (1) is thus replaced by

$$\boldsymbol{x}(t) = \sum_{\boldsymbol{k}} \boldsymbol{X'}_{\boldsymbol{k}}(t) \exp\left[j\left\{k_1[\omega_{10}t + \Phi_{\boldsymbol{R}}(t)] + \sum_{h=2}^{H} k_h\omega_h t\right\}\right] \tag{4}$$

where ω_{10} is the value that the free fundamental ω_1 takes on in the absence of modulation. Note that the time-dependent harmonics in (4) are denoted by $\boldsymbol{X'}_{\boldsymbol{k}}(t)$ and are related to the harmonics $\boldsymbol{X}_{\boldsymbol{k}}(t)$ used in the original representation (1) by

$$\boldsymbol{X}_{\boldsymbol{k}}(t) = \boldsymbol{X'}_{\boldsymbol{k}}(t)\exp[jk_1\Phi_{\boldsymbol{R}}(t)]. \tag{5}$$

When (4) is adopted, by definition, the phase of the reference harmonic $X'_{\boldsymbol{R}}(t)$ must be held fixed to zero throughout the analysis. As in the forced case [5], we may now replace the slowly modulated electrical regime (4) by an ordinary quasiperiodic regime in the neighborhood of each MS instant t_n. We assume that the "best" local approximation to a slowly modulated sinusoid is represented by an ordinary sinusoid whose amplitude, phase, and frequency coincide with the instantaneous amplitude, phase, and frequency of the modulated signal evaluated at the sampling instant of interest. In the neighborhood of t_n, (4) is thus replaced by

$$\sum_{\boldsymbol{k}} \boldsymbol{X'}_{\boldsymbol{k}}(t_n)\exp[jk_1\Phi_{\boldsymbol{R}}(t_n)] \exp\left[j\left\{k_1[\omega_{10} + \Delta\omega_{\boldsymbol{R}}(t_n)]t + \sum_{h=2}^{H} k_h\omega_h t\right\}\right] \tag{6}$$

where $\Delta\omega_{\boldsymbol{R}}(t) = d\Phi_{\boldsymbol{R}}(t)/dt$ is the instantaneous angular frequency deviation of the reference harmonic. Note that the harmonics of the local regime (6) coincide with the values that the time-dependent harmonics of the original representation (1) take on at $t = t_n$. In order to solve the local HB system by the Newton method for autonomous quasiperiodic regimes, the phase of an arbitrary harmonic, say $X_{\boldsymbol{R}}(t)$, must be fixed [7]. Since the phase of $X'_{\boldsymbol{R}}(t)$ must be zero for any t, according to (5), the phase of $X_{\boldsymbol{R}}(t_n)$ should be set to $\Phi_{\boldsymbol{R}}(t_n)$, because by definition $R_1 = 1$. This phase value is unknown. However, by integrating the frequency deviation from t_{n-M} to t_n we obtain

$$\begin{aligned}\Phi_{\boldsymbol{R}}(t_n) &= \Phi_{\boldsymbol{R}}(t_{n-M}) + \int_{t_{n-M}}^{t_n} \Delta\omega_{\boldsymbol{R}}(t)\,dt \\ &\approx \Phi_{\boldsymbol{R}}(t_{n-M}) + \sum_{m=0}^{M} c_m \Delta\omega_{\boldsymbol{R}}(t_{n-m})\end{aligned} \tag{7}$$

where the c_m are coefficients of a constant-step integration formula [6]. In order to correctly formulate the autonomous HB problem, we retain the basic structure of the nonautonomous system (3) and impose the phase constraint (7) by adding an auxiliary equation. Assuming that the real and imaginary parts of $X_{\boldsymbol{R}}(t_n)$ are the pth and qth entries of the real state vector $\boldsymbol{S}_n$, namely S_{np}, S_{nq}, the HB solving system (3) is replaced by (8), where the unknown fundamental frequency has been put into evidence for convenience

$$\begin{cases} \boldsymbol{E}_n[S_n,\, \omega_{10} + \Delta\omega_{\boldsymbol{R}}(t_n);\, \boldsymbol{S}_{n-1},\, \ldots,\, \boldsymbol{S}_{n-M}] = 0 \\ \sin[\Phi_{\boldsymbol{R}}(t_n)]S_{np} - \cos[\Phi_{\boldsymbol{R}}(t_n)]S_{nq} = 0. \end{cases} \tag{8}$$

Owing to the relationship (7) between $\Delta\omega_{\boldsymbol{R}}(t_n)$ and $\Phi_{\boldsymbol{R}}(t_n)$, for every n (8), is a system of $U = T + 1$ equations in as many unknowns. For $n = 1$, we have $\Delta\omega_{\boldsymbol{R}}(t_1) = 0$, $\Phi_{\boldsymbol{R}}(t_1) = 0$, so that the unknowns are the entries of $\boldsymbol{S}_1$ and ω_{10}. For $n > 1$, ω_{10} is held fixed, and the unknowns are the entries of $\boldsymbol{S}_n$ and $\Delta\omega_{\boldsymbol{R}}(t_n)$. The Jacobian matrix of (8) is

$$\boldsymbol{J} = \left[\begin{array}{c|c} \boldsymbol{J}_n & \dfrac{\partial \boldsymbol{E}_n}{\partial \omega_1} \\ \hline \boldsymbol{D} & D_1 \end{array}\right] \tag{9}$$

where $\boldsymbol{J}_n$ is the $T \times T$ Jacobian matrix of the forced system (3), $\partial\boldsymbol{E}/\partial\omega_1$ is a column vector of size T, and $\boldsymbol{D}$ is a $1 \times T$ row matrix. When (9) is multiplied by a real vector $\boldsymbol{g_U}$ of dimension U, most of the CPU time is spent in the multiplication of $\boldsymbol{J}_n$ by the first T entries of $\boldsymbol{g_U}$, which can be accomplished by the efficient algorithms discussed in [4] for the nonautonomous case. The overhead due to the additional equation appearing in (8) with respect to (3) is negligible when U exceeds a few thousand. This allows the use of Krylov-subspace techniques in conjunction with the envelope-oriented analysis of autonomous circuits with essentially the same numerical performance already demonstrated for completely forced circuits [5].

III. Applications

We consider a single-conversion transmitter front-end, including a doubly-balanced quadrature mixer arranged in a lower-sideband suppressing configuration, amplifiers, passive coupling circuits, and several linear parasitics, for a total of $n_D = 101$ device ports and 1050 nodes. The LO signal is sinusoidal with −6 dBm available power at 790 MHz. IM products of the LO and IF up to the 4th order are taken into account, so that $P = 20$. The signal source connected to the IF input is a VCO designed for a free-running fundamental frequency of oscillation $\omega_{10}/2\pi = 45$ MHz and an output power of 0 dBm when the modulating voltage is fixed to −5 V. The tuning sensitivity is about 300 kHz/V. We now assume that a digital signal consisting of an NRZ periodic sequence of 512 b with a bit rate of 80 kb/s is superimposed on the quiescent −5 V bias applied to the modulating input of the VCO. Each bit in the sequence is treated as a statistically independent random variable that may take on the values ±0.2 V with equal probabilities. The corresponding frequency deviation is ±60 kHz. The transitions between the two logical levels follow a raised-cosine waveform with 5%–95% rising and falling times equal to 1/10th of the bit interval. In Fig. 1, the actual signal spectrum at the front-end RF output is compared with the theoretical spectrum of the binary CPFSK signal generated by an ideal modulator fed by an infinite sequence of bits [8]. In Figs. 2 and 3, the instantaneous frequency deviation at the

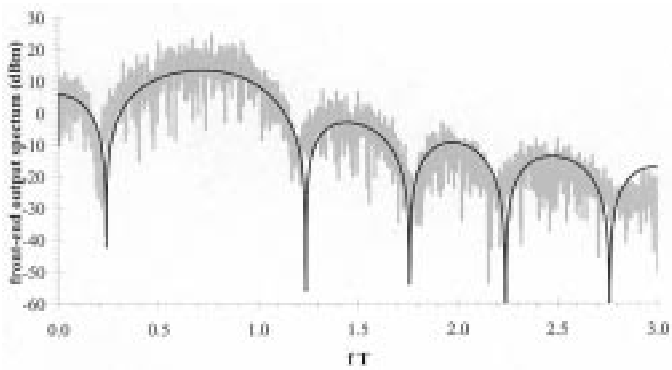

Fig. 1. Normalized spectrum of the front-end RF output signal compared with the spectrum of an ideal binary CPFSK signal (f = frequency offset from the carrier, T = bit interval).

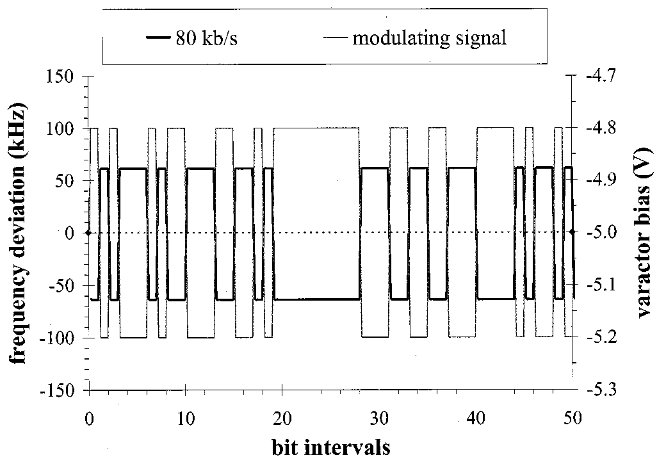

Fig. 2. Instantaneous frequency deviation of the front-end RF output signal compared with input modulating signal (80 kb/s). First 50 bits of a 512-bit sequence.

front-end RF output is compared with the modulating signal for two different bit rates (80 kb/s and 800 kb/s). The results clearly show the degradation of the modulation law at high bit rates. The analysis makes use of eight sampling points per bit, so that the total number of nodal unknowns is 176 332 800 (over 175 million). The simulation requires about 80 seconds of CPU time per MS instant, for a total of 327 680 s, and 387 MB of memory on an SUN Enterprise 450. Note that at the time of this writing Krylov-subspace HB analysis of autonomous circuits was not supported by commercial software. In order to give a clear feeling of the efficiency of the new analysis technique, the simulation is repeated for the frequency modulator only,

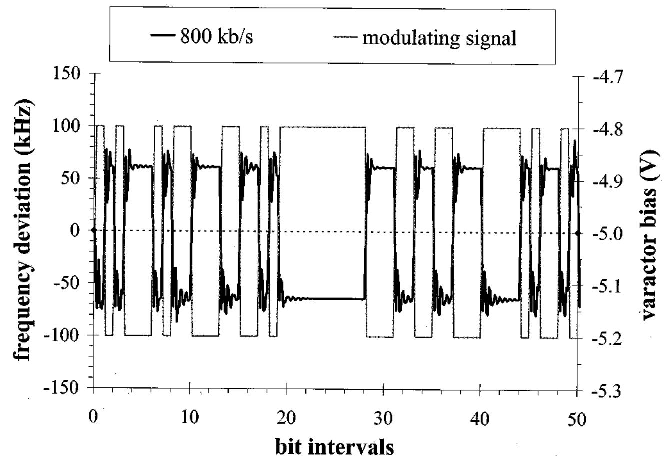

Fig. 3. Instantaneous frequency deviation of the front-end RF output signal compared with input modulating signal (800 kb/s). First 50 bits of a 512-bit sequence.

i.e., with the VCO output port loaded by a 50 Ω resistor. In this case, we have three device ports and 14 nodes, so that the total number of nodal unknowns reduces to 516 096. The simulation now requires about 0.1 s of CPU time per MS instant, for a total of 408 seconds. This is about nine times faster than a typical envelope-oriented technique based on the probe method for autonomous circuit analysis [3].

REFERENCES

[1] P. Feldmann and J. Roychowdhury, "Computation of circuit waveform envelopes using an efficient, matrix-decomposed harmonic balance algorithm," in *Proc. ICCAD*, 1996, pp. 269–274.

[2] V. Borich, J. East, and G. Haddad, "The method of envelope currents for rapid simulation of weakly nonlinear communication circuits," in *IEEE Microwave Theory Tech. Int. Microwave Symp. Dig.*, Anaheim, CA, June 1999, pp. 981–984.

[3] D. Hente and R. H. Jansen, "Frequency-domain continuation method for the analysis and stability investigation of nonlinear microwave circuits," *Proc. Inst. Elec. Eng. H*, vol. 133, pp. 351–362, Oct. 1986.

[4] V. Rizzoli *et al.*, "Fast and robust inexact-Newton approach to the harmonic-balance analysis of nonlinear microwave circuits," *IEEE Microwave Guided Wave Lett.*, vol. 7, pp. 359–361, Oct. 1997.

[5] V. Rizzoli *et al.*, "A Krylov-subspace technique for the simulation of RF/microwave subsystems driven by digitally modulated carriers," *Int. J. RF Microwave Comput.-Aided Eng.*, vol. 9, pp. 490–505, Nov. 1999.

[6] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1965.

[7] V. Rizzoli and A. Neri, "Harmonic-balance analysis of multitone autonomous nonlinear microwave circuits," in *IEEE Microwave Theory Tech Int. Microwave Symp. Dig.*, Boston, MA, June 1991, pp. 107–110.

[8] S. Benedetto, E. Biglieri, and V. Castellani, *Digital Transmission Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1987.